Children's Representational Choices and Preferences during Word Problem Solving

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Abstract

- The current study investigates children's self-selected repertoire for representing their during word-problem solutions. Participating children completed a series of story problems to evaluate their problem-solving behaviors and abilities. The story problems involve grade-level addition, subtraction, multiplication, and division problems. The results for first- and second-grade students showed a high overall rate of external representation use and differences in type of external representations, both within a grade cohort and comparing the first and second grade. The differences follow a predicted shift from more concrete representations to more abstract. Most participants, especially in the second-grade cohort, used more than one type of external representation, suggesting familiarity and confidence in the use of that representation type.

Background

- An assumption embedded within common mathematics education practices, and with roots in both Piagetian and Vygotskian perspectives on development, is that children should be aided to move from concrete representations of quantities to more abstract (National Council of Teachers of Mathematics, 2000), especially in the early childhood and early elementary years (Ball, 1992).
- Learning mathematics can entail not merely *dual* representations (e.g., DeLoache, 1995), but multiple forms of mathematical expression (Gentner & Ratterman, 1991). Much is yet to be understood about how children use and select these symbols.
- Offering a choice of materials (concrete counters and blocks, paper and pencil for drawings, tallies, or other written symbols, a 100 chart, and a number line) to express their ideas creates the opportunity for children to use any one of the available tools or none or to use multiple representations (e.g., Fennell & Rowan, 2001) to express their answer and for the researcher to study children's choice and use of multiple symbolic forms.
- The open-ended story problems used, modeled after the Cognitively Guided Instruction program (Carpenter et al., 1999) do not specify a solution strategy, facilitating examination of children's strategic thinking (e.g., Verschaffel & Decorte, 1993).

Method: Participants

- Participants all attended the same suburban Midwestern primary school and reflected the diversity of that school.
  - 20 kindergartners
  - 26 first graders (although 4 moved out of the school district over the summer and did not participate in the delayed follow-up session)
  - 30 second graders participated in the sessions.
- The results below come from the sessions for 15 first-grade students (average age at first test = 88 months; 7 boys and 8 girls) and 16 second-grade students (average age at first test = 98 months; 7 boys, 9 girls).
- All children attended the same school and received instruction with the same National Council of Teachers of Mathematics reform-based curriculum and thus could all be assumed to have some familiarity with word problems.

Method: Procedure

- A longitudinal, cross-sectional, quasi-microgenetic (e.g., Siegler, 2002) design was used.
- Students participated in 3 sessions: the first two approximately one month apart in Spring, with a delayed follow-up session in late autumn. Each child answered 6 grade-level word problems (Carpenter et al., 1999) per session.
  - Students participated in individual videotaped sessions.
- The Time 2 problems paralleled those from Time 1 except for the numbers used; the Time 2 problems were used again at Time 3 (delayed follow-up).
- The experimenter read each problem two times and repeated it as many times as requested. After time to solve the problem, either the child volunteered and answer or the experimenter prompted for one and then asked, “Can you tell me how you figured that out?”.
- *Materials.*
  - Prior to beginning the problem solving, the experimenter told the participant: “I brought some math tools that you can use if you’d like. There’s are paper and pencil and markers if you want to write anything or draw anything, there’s 100 charts, there’s a number line that goes from 0 to 25, and three types of blocks: the base-ten pieces, the unifix cubes, and the wooden cubes. [See photographs of examples] You can use any of these if they help you solve the problem, but you don’t have to. If you don’t want to use any of them, that’s fine too. It’s totally up to you. Solve the problems in whatever way makes the most sense to you.”
  - The materials were all placed within the child’s reach but the experimenter never encouraged or prompted a child to use them. The selection of available materials corresponds to the tools available in typical early elementary instruction.
- *Coding.*
  - The use of external representations was coded from video, noting the use or creation of any representation during problem solving and subsequent explanation.
  - Gesture was credited when a student counted on fingers or held up fingers to keep track of counting or during the explanation.
  - On-paper representations were categorized as either child-created number expressions (e.g., 43- 17 = 26) and drawings (including tally marks or more iconic drawings, such as a tree with birds in it) or use of the preprinted two-dimensional representations, either the number line or the 100 chart.
  - Use of concrete manipulatives was credited for any non-play use of the blocks during problem solving.

Results: How many problems did children get correct, by grade and session?

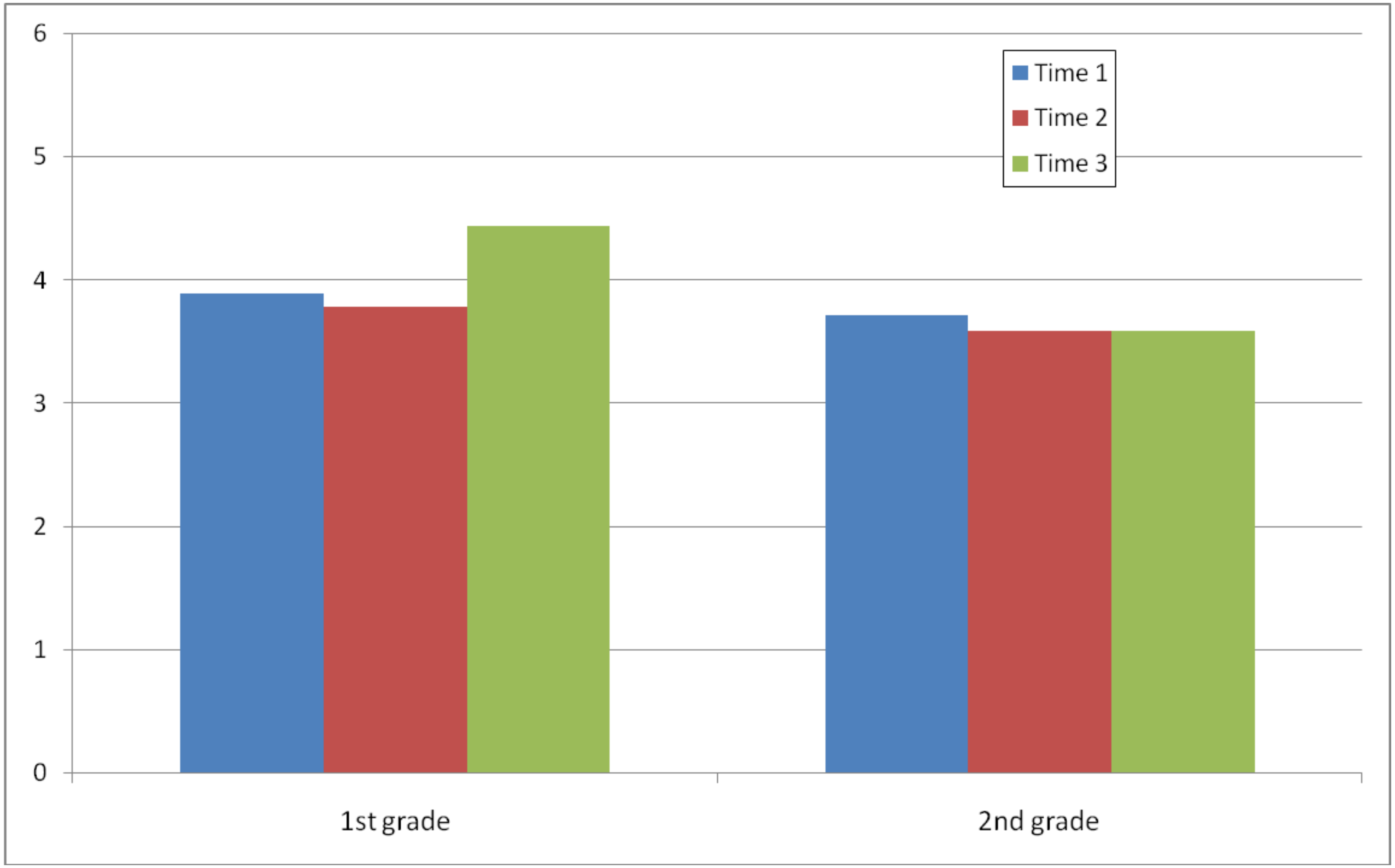
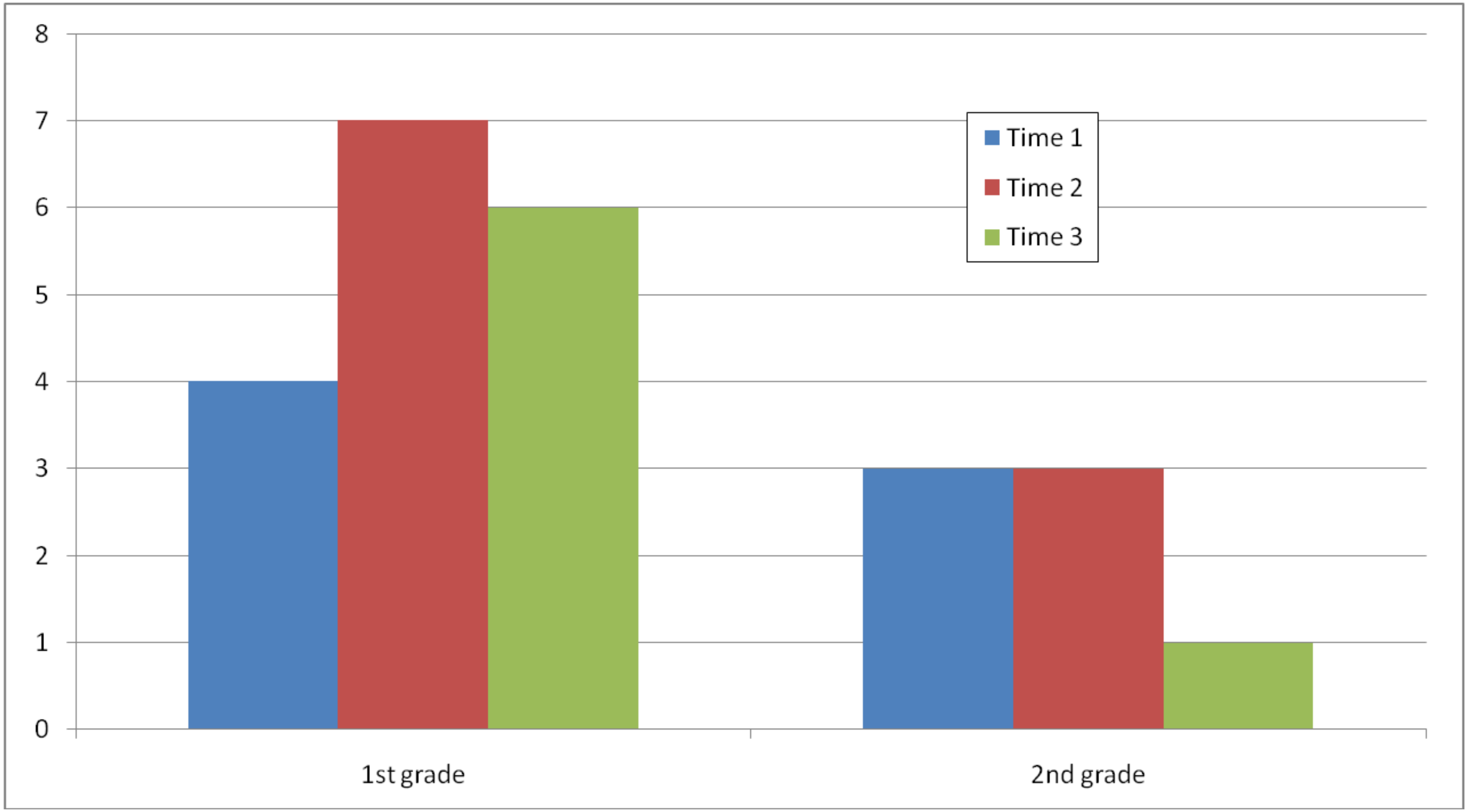


Figure 1: Average number of problems correct by grade cohort and session, out of 6. No significant differences were observed.

How many children used only one type of external representation for a 6-problem set?



- Figure 2: Number of children per session who used only one type of representation during a session (first grade, n = 15; second grade, n =16). Rates did not differ by time period or grade. Additionally, this provides an index that children, especially those in the second-grade cohort, used multiple external representations rather than favoring one representational type.

How often by session and grade did children use external representations from each of the categories?

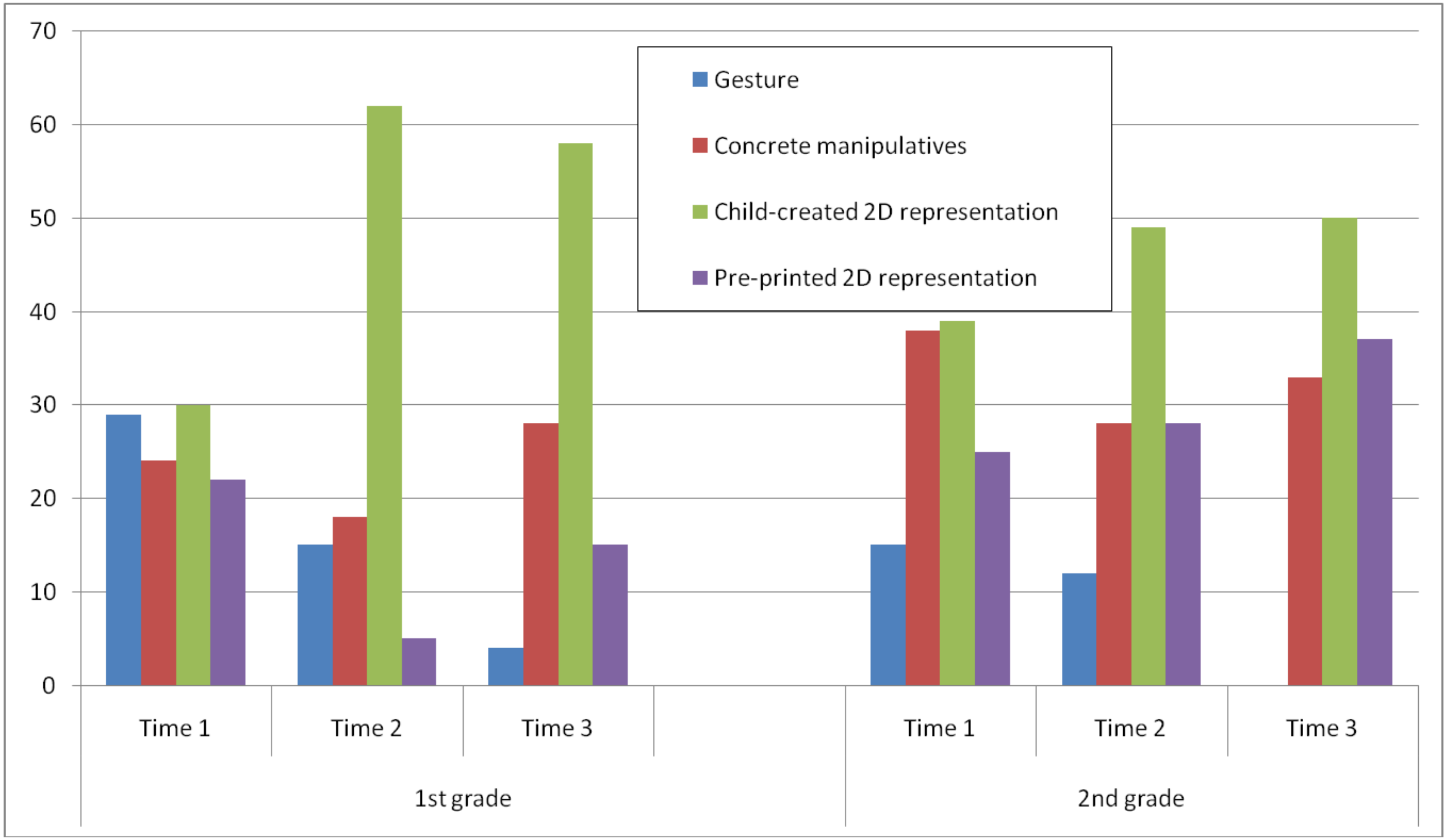


Figure 3: Number of problems on which representation types were used.

- Note that the first-grade set included 6 problems for 15 children at each time point (and thus 90 problems), while the second grade included 6 problems for 16 children at each time point (and thus 96 problems).
- Examining the children's use of fingers, concrete representations, student-created drawings/writing, and preprinted numeral representations, the majority of children in both grades, for both sessions, used a symbolic representation to solve all problems differ. Every child used or created a symbol on at least one of the problems in each session. Participants in both grades decreased their use of gesture over the sessions and significantly increased their rate of drawings or written equations. Second-grade participants significantly increased their use of pre-printed 2D materials, particularly the 100 chart, perhaps stemming from increased classroom experiences with this form of number sequence representation for arithmetic problem solving.

How often did students opt to use more than one external representation (e.g., concrete manipulatives and the 100 chart) on any single problem?

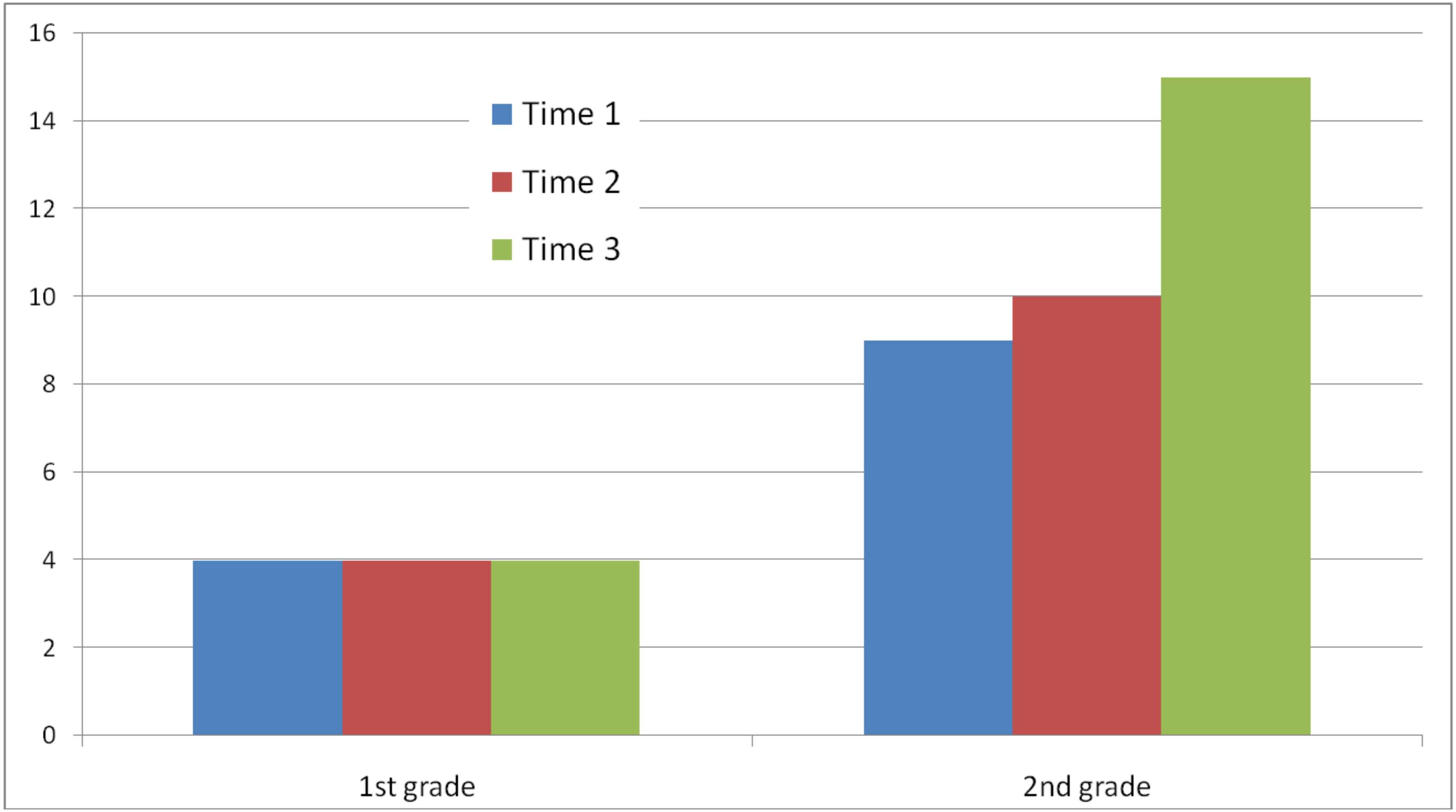


Figure 4: Multiple external representations within a problem

- Figure 4: Multiple external representations within a problem. Significantly more 2nd graders used multiple external representations for any single problem than 1st graders.
- Note that although 4 children used multiple external representations on at least one problem at Time 1, Time 2, and Time 3, these were not necessarily the same children. Across the three sessions, 8 different students used more than one external representation on a single problem. Similarly, all but 2 of the 2nd graders used a combination of external representations on at least 1 problem. This result provides evidence that this cohort of 2nd graders may have had more confidence and fluency in coordinating multiple external representations.

How many children, by grade and session, opted for more than one category of external representation during a problem-solving session, although not necessarily on any one problem?

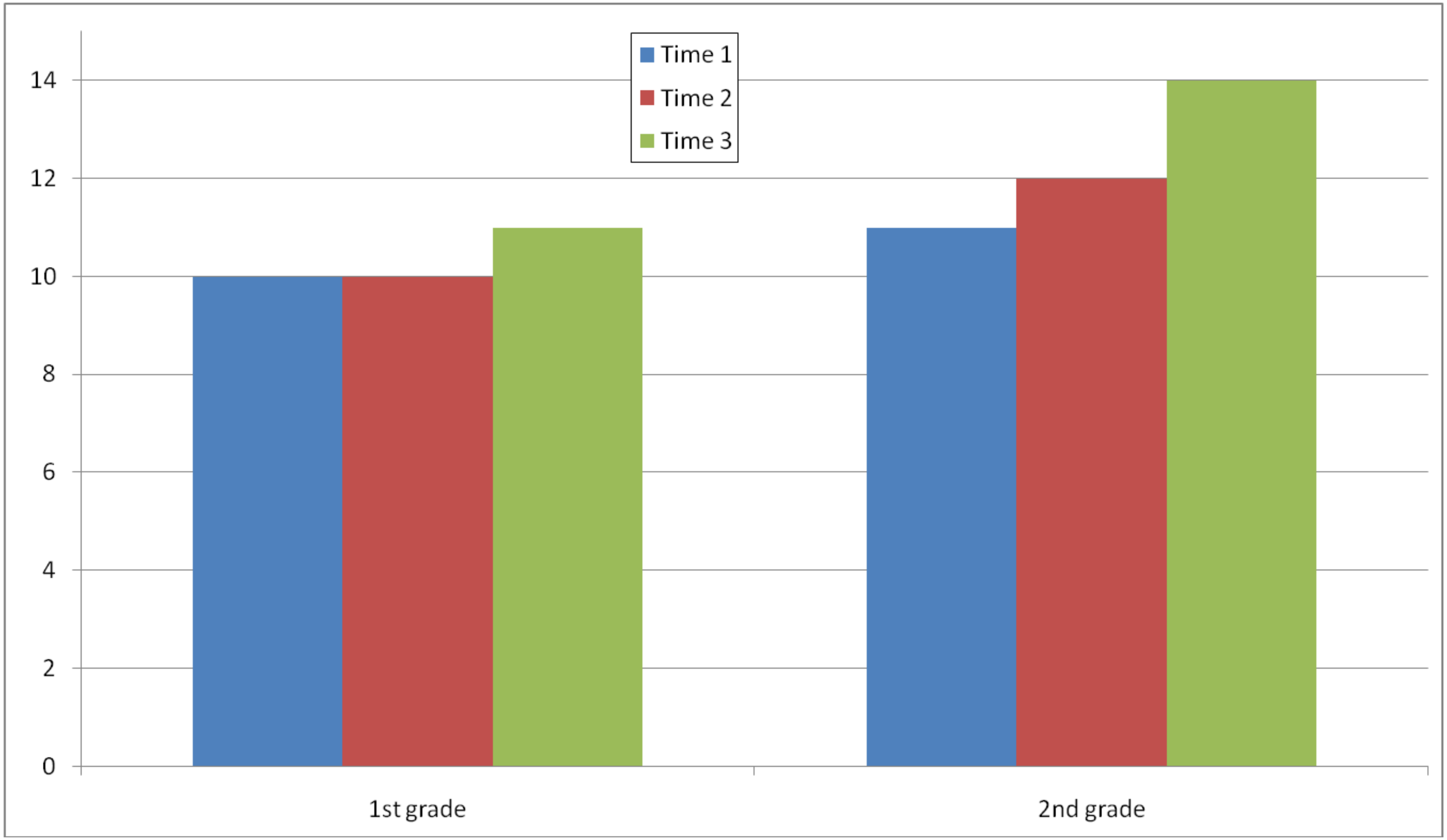


Figure 5: The rate by grade and session of children using more than one representation

- Figure 5: The rate by grade and session of children using more than one representation used during a session, not necessarily on the same problem. Parallel to the finding for the 1st grade children's use of multiple representations on a single problem, many 1st graders used multiple external representations within one session but not necessarily all three. Only 2 1st graders did not use a multiple external representations during at least 1 session. Similarly, for the 2nd grade, only 1 student did not use more than one representation types across the 6 problems.
- Examining these results together, a subset of children from each grade could be considered consistently eclectic in their use of external representations, using multiple forms across the 6 problems for a session, but many of their peers used multiple external representations only occasionally, raising the question of whether their use of multiple representations was situational (e.g., induced by a struggle on a particular problem), rather than a characteristic preference for the child.

Discussion

- The children thus spontaneously opted to use external representations, but displayed differences in the type and frequency of their use, and changes from session to session in favor of creating two-dimensional representations and referring to manipulatives less. These results recall a hypothesized evolution of children's symbol use in mathematics from concrete to abstract (e.g., Carpenter et al., 1999). Given that signs and symbols may mediate learning (e.g., Uttal, 2000), this investigation addresses children's preferences for symbolizing quantities and has potential implications for instructional interventions.
- The use of concrete objects to stand for quantities does not guarantee that a child will identify the intended meaning of these tools (e.g., Baroody, 1989, Clements, 1999). Given the widespread use of concrete objects as problem solving and problem representation tools, it is important to document how individual learners understand and use these objects. Then, even given successful use of manipulatives for problem solving, the mapping to more abstract representations, such as writing an equation, cannot be assumed to be an easy or transparent process for a young learner. As noted by Uttal, Scudder, and DeLoache (1997) "Connections must be established for both numerical symbols and operation symbols." (Wearne & Hiebert, 1988, p. 372)" (p. 48). Young learners' mappings or translations between representations must be supported.

Educational Implications

- Teachers should offer explicit scaffolding in the use of external representations to increase the likelihood that the representation will be used as intended (e.g., Uttal, Scudder, & DeLoache, 1999). Teachers or other adults cannot take the intended symbol mappings as transparent for the children (e.g., DeLoache, 1995; Hughes, 1986), and by observing their representation choices can gain insight into which external representations students find the most representationally clear.
  - Anecdotally, children often became confused about how to move on a number line or use the 100 chart for computation, e.g., moving by 10 on the chart when the problem called for multiplication by 8.
- Educators' observation and assessment of learners' preferences for types of external representations can facilitate the scaffolding of their use of other types and recognition of the similarity in intended referents of that other representation type. In particular, when moving from concrete to abstract, learners need to be scaffolded to see the relations between elements such that, for example, the ten base-ten block maps onto a line drawn on paper while the unit piece maps onto a dot drawn on paper.
- In this era of increased testing of school students, it is imperative that we identify the most effective and efficient means of evaluating children's essential knowledge.

Examples of Children's Spontaneous Comments about External Representations

- 2nd grader EY: "This is how we learned to do it in class. A line stands for ten and a dot stands for one and that way we don't have to draw them out like the base-ten blocks (…) Sometimes it's hard to keep track when I trade a ten for ones."
- 1st grader CD on using the 100 Chart, "Ugh, I forget which number I started on."
- 2nd grader JM: "I don't use any of that stuff. I just do it in my head." but then he went on to use both written and concrete representations on some problems. On the ones that he solved only in his head, he had some difficulty keeping track of his numbers.
- 2nd grader AO on the base-ten blocks: "These are easier than those (the unifix snapcubes) because those you have to count by one, but these you can just take 1 piece and say "10"."
- 1st grader NC: "I think I'm going to use these (the unifix connecting cubes) cubes right here because they're easiest to work with."
- 1st grader BH, working with paper and pencil: "This is hard." E: Can you think of a different way to figure it out? BH: Yes! (takes out unifix cubes) (...) I knew it was wrong and I thought the cubes might help.

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Appendix: Sample Problems

- 1st grade:
  - Part-part-whole (part unknown): Shelly had 16 jellybeans. Some are red and the rest are blue. 7 of the jellybeans are blue. How many jellybeans are red?
- 2nd grade
  - Compare (quantity unknown): Matt has 23 comic books. Ryan has 18 more comic books than Matt. How many comic books does Ryan have?